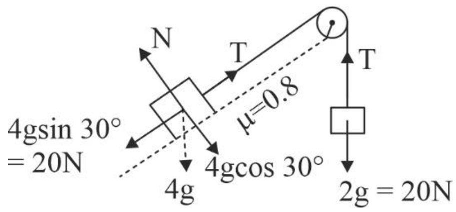
Solutions of Mock JEE Advanced-4 (CBT) | Paper - 1 | 2024

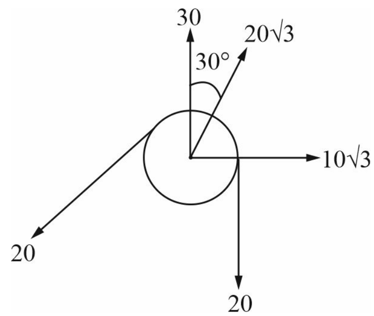
PHYSICS

SECTION - 1

1.(ABCD)

Two blocks system will not have tendency to slide.





$\therefore \quad fr = 0$

Acceleration, $a = 0$

$\therefore \quad T = 20N$ $\qquad \therefore \quad \vec{F} = 10\sqrt{3}\hat{i} + 30\hat{j}$

Only contact force on the block by inclined plane will be the normal force. $(\because fr = 0)$

$N = 4g\cos 30° = 20\sqrt{3}N$

2.(ABCD) Fringe width, $\beta = \frac{D\lambda}{d}$

With decrease in D, fringe width (β) will decrease.

Hence, all fringes will come nearer to central line on screen.

Distance of 2nd bright fringe

$y_2 = \frac{2D\lambda}{d} \qquad \therefore \qquad \frac{d(y_2)}{dt} = \frac{2\lambda}{d}\left(\frac{dD}{dt}\right) = \frac{2\lambda}{d}(-v)$

$\therefore$ Speed of 2nd bright fringe with respect to screen will be $\frac{2\lambda v}{d}$

3.(ACD) $\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{0.2\times10^{-3}\times2\times10^{-6}}} = 5\times10^4 rad/s$

At $t = 0$; charge on both capacitors is same.

$Q_0 = C_{eqv.}V = \left(\frac{2\times2}{2+2}\right)\times 20\mu C$ (series combination)

$= 20\mu C$

Max energy stored in capacitor C_1 = max energy stored in inductor

$\frac{q_{max}^2}{2C_1} = \frac{1}{2}Li_{max}^2$

$\Rightarrow \quad \frac{(20\times10^{-6})^2}{2\times2\times10^{-6}} = \frac{1}{2}\times(0.2\times10^{-3})i_{max}^2$ $\quad [q_{max} = Q_0 = 20\ \mu C] \quad \Rightarrow \quad i_{max} = 1A$

Charge on capacitor C_1 after key K_1 is opened and K_2 is closed.

$q = Q_0 \sin(\omega t + \phi)$

At $\quad t = 0, \quad q = Q_0$

$\therefore \quad Q_0 = Q_0 \sin\phi \Rightarrow \phi = \frac{\pi}{2} \qquad \therefore \quad q = Q_0 \sin\left(\omega t + \frac{\pi}{2}\right)$

$q = Q_0 \cos(\omega t)$

$\therefore$ Current through inductor at any time $t = t$, $\quad i = \frac{dq}{dt} = -Q\omega \sin\omega t$

At time when;

Energy in inductor $= \frac{1}{3} \times$ Energy in capacitor C_1

$$\frac{1}{2}Li^2 = \frac{1}{3} \times \frac{q^2}{2C_1}$$

$$\Rightarrow \quad \frac{1}{2}L \times \left(-Q_0\omega \sin\omega t\right)^2 = \frac{1}{3} \times \frac{1}{2C_1} Q_0^2 \cos^2(\omega t) \qquad \left[\omega^2 = \frac{1}{\sqrt{LC_1}}\right]$$

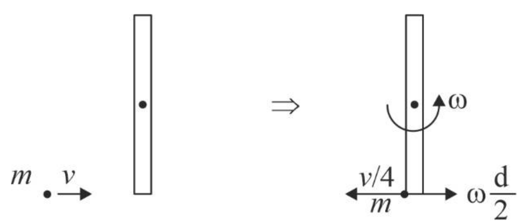
$$\Rightarrow \quad \tan(\omega t) = \frac{1}{\sqrt{3}} \qquad \Rightarrow \quad \omega t = \frac{\pi}{6}$$

$\therefore \quad$ Charge on capacitor at this time

$$q = Q_0 \cos\omega t = 20 \times \cos\left(\frac{\pi}{6}\right) = 10\sqrt{3}\mu C$$

SECTION-2

4.(B)



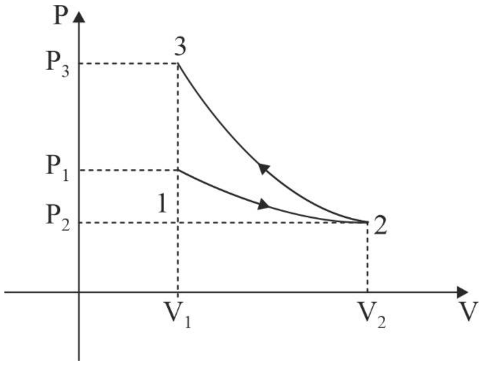
From conservation of angular momentum about the pivot,

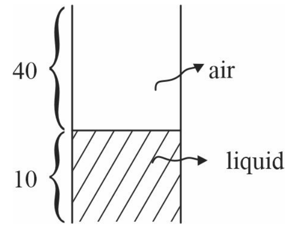
$$mv.\frac{d}{2} = -m\frac{v}{4} \times \frac{d}{2} + \frac{Md^2}{12}.\omega \quad \text{Or,} \quad \frac{6md^2}{12}.\omega = \frac{5}{8}.mvd$$

$$\Rightarrow \quad \omega = \frac{5v}{4d} \qquad \ldots(1)$$

$$e = \frac{\text{Relative velocity of separation along the line of impact}}{\text{Relative velocity of approach along the line of impact}} = \frac{\frac{\omega d}{2} + \frac{v}{4}}{v} = \frac{\frac{5v}{8} + \frac{v}{4}}{v} = \frac{7}{8}$$

5.(C) The two processes are shown in the following P-V diagram





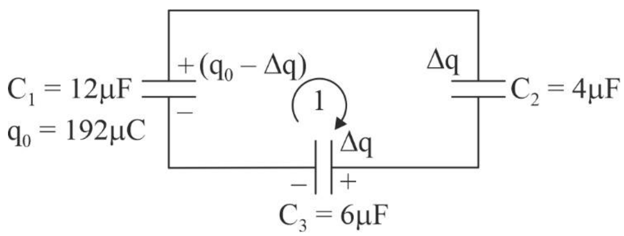
P-V-graph for adiabatic process is steeper than that for isothermal process.

From the graphs for the processes,

It is clear that $P_3 > P_1$ and $W < 0$

6.(C) In the close loop,

$$\frac{q_0 - \Delta q}{C_1} - \frac{\Delta q}{C_2} - \frac{\Delta q}{C_3} = 0$$



$$\frac{q_0}{C_1} = \Delta q\left[\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right] \quad ; \quad \Delta q = \frac{q_0}{C_1\left[\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right]} = \frac{192}{[1+3+2]} = 32\mu C$$

7.(A) $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$

Differentiating the expression ;

$$\frac{-dR}{R^2} = \frac{-dR_1}{R_1^2} - \frac{dR_2}{R_2^2} \Rightarrow \frac{\Delta R}{R^2} = \frac{\Delta R}{R_1^2} + \frac{\Delta R}{R_2^2} \Rightarrow = \frac{0.2}{25} + \frac{0.1}{100} = \frac{0.9}{100}$$

$$\Rightarrow \frac{\Delta R}{R} = \frac{0.9}{100} \times \frac{10}{3} = 0.03 \qquad \left[\because \quad R = \frac{10\times5}{10\times5} = \frac{10}{3}\Omega\right]$$

Now, $H = \frac{V^2}{R}t$

$$\therefore \quad \frac{\Delta H}{H} = \frac{2\Delta V}{V} + \frac{\Delta R}{R} + \frac{\Delta t}{t} = \left[2\times\frac{0.5}{20} + 0.03 + \frac{0.2}{10}\right]\times 100 = 10\%$$

SECTION-3

8.(C) $^{238}_{92}U \rightarrow ^{206}_{82}Pb + 8^4_2He + 6\ ^{0}_{-1}e$

$^{226}_{89}Ac \rightarrow ^{226}_{88}Ra + ^{0}_{+1}\text{e}$

$^{226}_{88}Ra \rightarrow ^{206}_{82}Pb + 5^4_2He + 4\ ^{0}_{-1}e$

$^{239}_{92}U \rightarrow ^{239}_{94}Pu + 2\ ^{0}_{-1}e$

Option (C) is correct.

9.(D) Resonant frequently

$$\omega_R = \frac{1}{\sqrt{LC}} = \sqrt{\frac{1}{10\times10^{-3}\times10^{-6}}} = 10^4 rad/s$$

10% less frequency of above resonant frequency is

$$\omega = 10^4 - 10^4 \times \frac{10}{100} = 9\times10^3 rad/s$$

$$X_L = \omega \times L = 9\times10^3\times10\times10^{-3} \Rightarrow 90\,\Omega$$

$$X_C = \frac{1}{\omega C} = \frac{1}{9\times10^3\times1\times10^{-6}} = \frac{10^3}{9} = 111.1$$

$$Z = \sqrt{R^2 + (X_C - X_L)^2}\ ;\ = \sqrt{(3)^2 + (111.1-90)^2} = 21.32$$

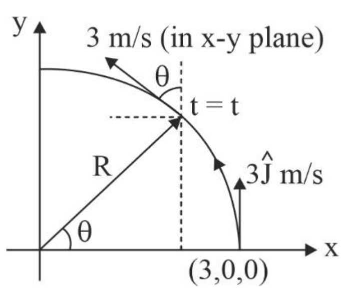
(P) Current amplitude $I_0 = \frac{V_0}{Z} = \frac{15}{21.32} = 0.704$

(R) Power factor $\cos\phi = \frac{R}{Z} = \frac{3}{21.32} = 0.141$

(Q) Average power $P = \frac{1}{2}V_0 I_0 \cos\phi = \frac{1}{2}\times15\times0.704\times0.141 = 0.744$

(S) $Q = \frac{X_L}{R} = \frac{\omega_R L}{R} = \frac{10^4\times10\times10^{-3}}{3} = 33.3$

10.(C)



Radius of Helical path $R = \frac{mv_\perp}{B_0 q} = 1\times3 = 3m$

$T = \dfrac{2\pi m}{qB_0} = 2\pi \sec,$ pitch $= 4 \times 2\pi = 8\pi$ meter

Angle Rotated by charge particle

$$\theta = \frac{2\pi}{\left(\dfrac{2\pi m}{qB_0}\right)}.t = \left(\frac{qB_0}{m}\right)t = t$$

$\therefore$ At any time t = t;

$\vec{r} = 3\cos t\,\hat{i} + 3\sin t\,\hat{j} + 4t\hat{k}$

$\vec{v} = -3\sin t\,\hat{i} + 3\cos t\,\hat{j} + 4\hat{k}$

$\vec{a} = -3\cos t\,\hat{i} - 3\sin t\,\hat{j}$

11.(D) $K = 4\pi \Rightarrow \lambda = \dfrac{2\pi}{k} = 1/2$

Positions of displacement Nodes/Pressure Antinodes: $0, \dfrac{\lambda}{2}, \lambda, \dfrac{\lambda}{2}, 2\lambda, \dfrac{5\lambda}{2}, 3\lambda......$

$= 0, \dfrac{1}{4}, \dfrac{1}{2}, \dfrac{3}{4}, 1, \dfrac{5}{4}, ...$

Positions of displacement Antinodes / Pressure Nodes: $\dfrac{\lambda}{4}, \dfrac{3\lambda}{4}, \dfrac{5\lambda}{4}, \dfrac{7\lambda}{4}, ...$

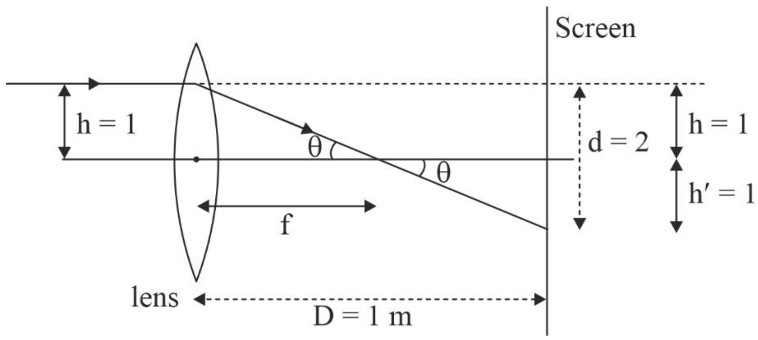
= 1/8, 3/8, 5/8, 7/8, ...

$P \rightarrow 1, 2$; $Q \rightarrow 3, 4$; $R \rightarrow 3, 4$; $S \rightarrow 1, 2$

Option (D) is the best matching.

SECTION-4

1.(5)

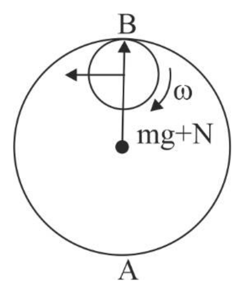


From given graph,

For $h = 1, d = 2$; $h' = d - h = 2 - 1 = 1$

$$\tan\theta = \frac{h}{f} = \frac{h'}{(D-f)} \Rightarrow \frac{1}{f} = \frac{1}{(1-f)} \Rightarrow f = 0.5\,\text{m} = 50\text{ cm}$$

2.(9) At top print (B),



$$mg + N = \frac{mV_{c.m,B}^2}{(R - R/4)} \qquad (\text{N} = 0)$$

$$\Rightarrow \quad V_{c.m,B} = \sqrt{g \times \left(\frac{3R}{4}\right)}$$

$$\therefore \quad \omega_B = \frac{V_{c.m}}{R/4} = \frac{\sqrt{\frac{3}{4}gR}}{R/4} = 2\sqrt{\frac{3g}{R}}$$

From energy conservation between top and bottom points of hoop;
Loss in G.P.E. = Gain in K.E.

$$mg \times \left(\frac{6R}{4}\right) = \left(\frac{1}{2}mV_{c.m_A}^2 + \frac{1}{2}I_{c.m.}\omega_A^2\right) - \left(\frac{1}{2}mV_{c.m_B}^2 + \frac{1}{2}I_{c.m.}\omega_B^2\right)$$

$$= \frac{1}{2}\left\{m\left(\frac{R}{4}\right)^2 + m\left(\frac{R}{4}\right)^2\right\}\left[\omega_A^2 - \omega_B^2\right] = \frac{mR^2}{16}\left(\omega_A^2 - \frac{12g}{R}\right)$$

$$\Rightarrow \quad \frac{24g}{R} = \omega_A^2 - \frac{12g}{R} \quad \Rightarrow \quad \omega_A^2 = \sqrt{\frac{36g}{R}} = 2\sqrt{\frac{9g}{R}}$$

3.(4) Induced EMF = $E(2\pi r) = \dfrac{d\phi}{dt}$ $\qquad \Rightarrow E = \dfrac{\pi a^2 2B_0 t}{2\pi r}$

Torque due to field about centre of ring $\tau_1 = (qE)r = (\lambda(2\pi r))\left(\dfrac{2\pi a^2 B_0 t}{2\pi r}\right)r$

Ring starts rotating, when τ due to electric field = τ due to friction.

$\tau_1 = (\mu mg)r$ Solving $t = \dfrac{\mu mg}{2\pi a^2 B_0 \lambda} = 4s$.

4.(4) $\Delta m.c^2 + K_A + K_B = K_C$ + excitation energy

$\left(m_A + m_B - m_C\right)c^2 + K_A + K_B = K_C$ + excitation energy ; c = speed of light

Putting the values ; $K_C = 2\ MeV$

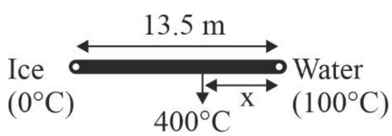
5.(9) $\frac{2}{3}\ell_1 = \frac{3}{4}\ell_2 \Rightarrow \frac{\ell_1}{\ell_2} = \frac{9}{8}$; l_1 and l_2 are the distances between stars in given two binary star systems

$$\frac{G(2M)(M)}{l_1^2} = \frac{MV_{0,1}^2}{r} \qquad \Rightarrow V_{0,1} \propto \frac{\sqrt{(2M)r}}{l_1}$$

$$T_1 = \frac{2\pi r}{V_{0,1}} \propto \frac{l_1}{\sqrt{2M}} \qquad [r \to \text{same in both cases}]$$

Similarly, $T_2 \propto \frac{l_2}{\sqrt{3M}}$ $\quad \therefore \quad \frac{T_1}{T_2} = \frac{l_1}{l_2}\sqrt{\frac{3M}{2M}} = \frac{9}{8}\sqrt{\frac{3}{2}}$

6.(135)



$$Q_{ice} = KA\left(\frac{400-0}{13.5-x}\right)t$$

So, ice melt, $m_{ice} = \frac{Q_{ice}}{L_f} = \frac{KA(400-0)t}{80(13.5-x)}$ …(i)

$$Q_{\text{water}} = KA\frac{(400-100)}{x}t$$

$m_{\text{steam}} = \frac{Q_{\text{water}}}{L_V} = \frac{KA(400-100)}{540x}t$ …(ii)

$$m_{\text{ice}} = m_{\text{steam}}$$

$$\frac{400}{80(13.5-x)} = \frac{300}{540x}$$

$$6(13.5-x) = 54x$$

$$6\times 13.5 = (54x + 6x)$$

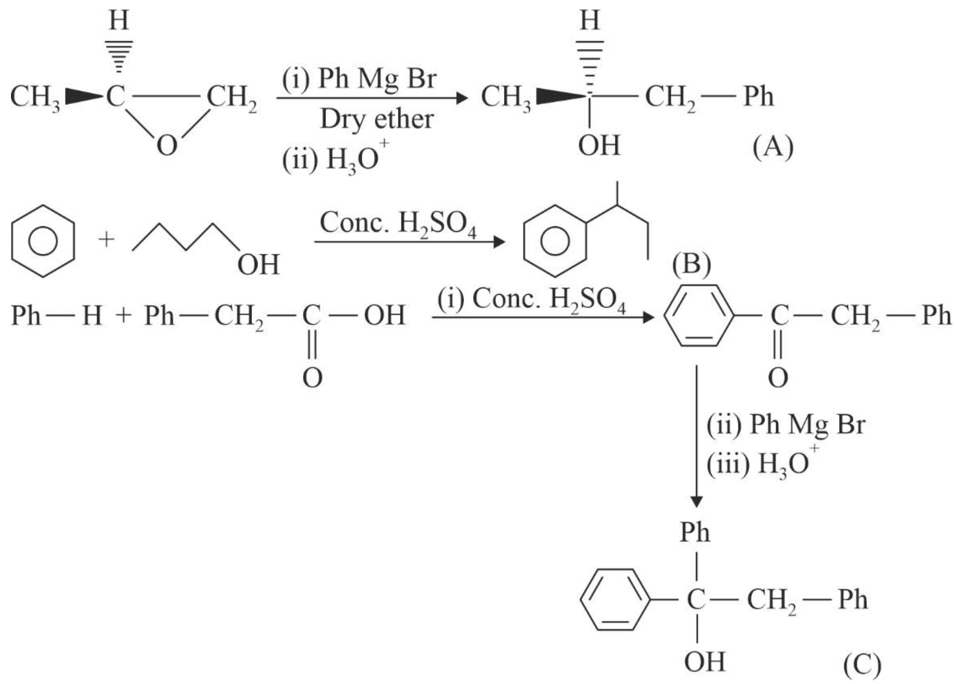
$$x = \frac{6\times 13.5}{60} = 1.35m$$

CHEMISTRY

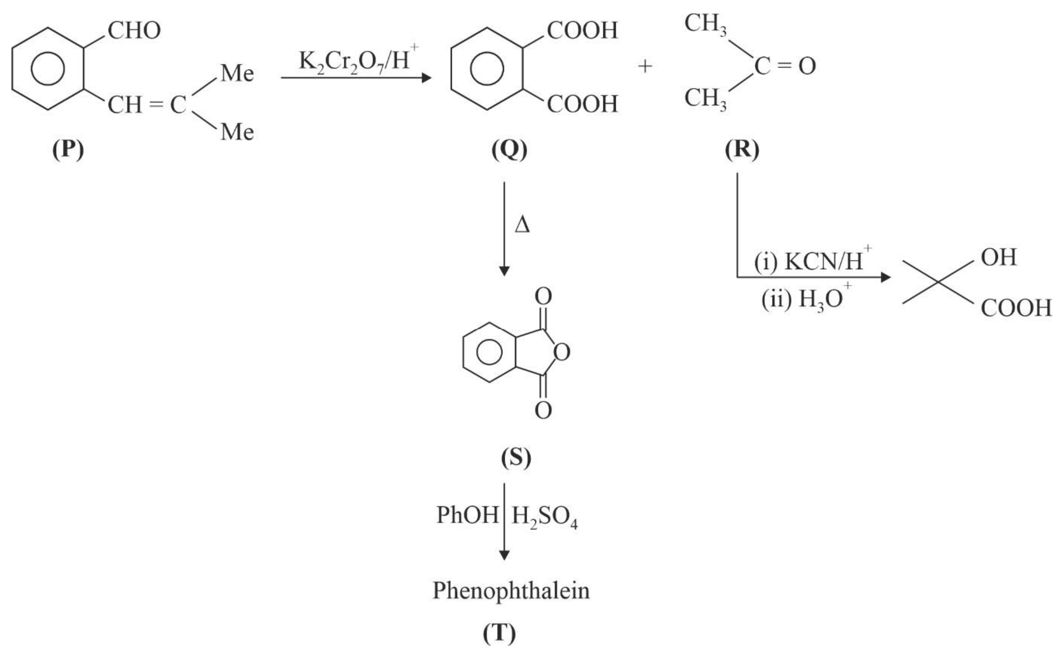
SECTION - 1

1.(ABC) Slag formed during extraction of iron is $CaSiO_3$ and Copper is $FeSiO_3$.

2.(AB)



3.(C)



SECTION - 2

4.(B) $\underset{\substack{(A)\\(\text{white})}}{ZnS} + H_2SO_4 \rightarrow \underset{(C)}{ZnSO_4} + \underset{(B)}{H_2S}$

$$3H_2S + K_2Cr_2O_7 + 4H_2SO_4 \rightarrow K_2SO_4 + Cr_2(SO_4)_3 + \underset{(D)}{3S} + 7H_2O$$

$$\underset{(D)}{S} + O_2 \rightarrow \underset{(E)}{SO_2}$$

$$2H_2S + SO_2 \rightarrow 2H_2O + 3S$$

$$ZnSO_4 + 2NaOH \rightarrow Zn(OH)_2 + Na_2SO_4$$

$$Zn(OH)_2 + 2NaOH \rightarrow \underset{(\text{Soluble})}{Na_2ZnO_2} + 2H_2O$$

5.(A) Equation of line $\Rightarrow \dfrac{1}{\Lambda_m} = \dfrac{1}{K_a \Lambda^{\circ 2}_m}(c\Lambda_m)$

Slope $= \dfrac{5}{16} = \dfrac{1}{K_a(\Lambda^\circ{}_m)^2}$; $K_a = \dfrac{16}{5 \times (400)^2} = 2 \times 10^{-5}$; $K_a = \dfrac{c\alpha^2}{1-\alpha}$

$2 \times 10^{-5} = 0.1 \times \alpha^2$; $1.4 \times 10^{-2} = \alpha$

6.(A) The minimum $\left[OH^-\right]$ at which there will be no precipitation of $Mg\left(OH\right)_2$ can be obtained by

$K_{sp} = \left[Mg^{2+}\right]\left[OH^-\right]^2 \Rightarrow 18.0 \times 10^{-12} = (0.1) \times \left[OH^-\right]^2 \quad \therefore \left[OH^-\right] = 1.34 \times 10^{-5} M$

Thus, solution having $\left[OH^-\right] = 1.34 \times 10^{-5} M$ will not show precipitation of $Mg\left(OH\right)_2$ in 0.1 M Mg^{2+}. These hydroxyl ions are to be derived by basic buffer of NH_4Cl and NH_4OH.

$$pOH = pK_b + \log\frac{[\text{Salt}]}{[\text{Base}]}$$

$$\Rightarrow pOH = pK_b + \log\frac{[NH_4^+]}{[NH_4OH]}$$

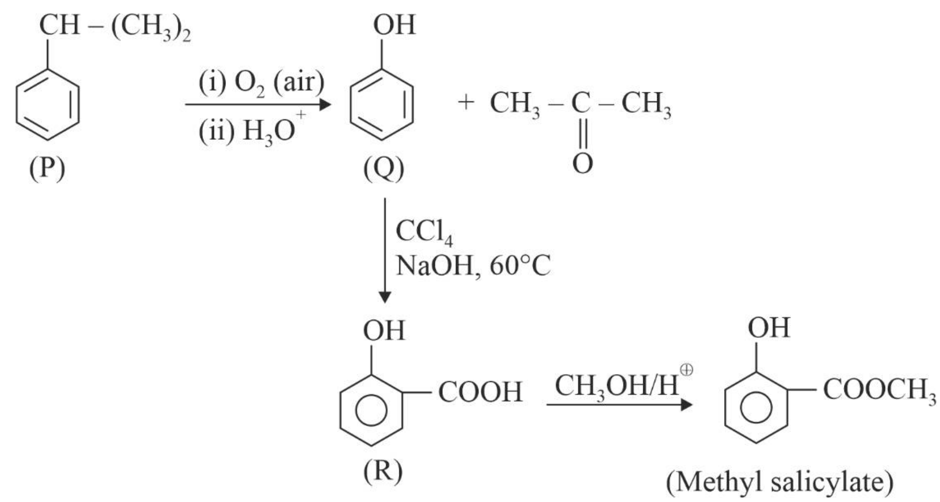
$$NH_4OH \rightleftharpoons NH_4^+ + OH^-$$

In presence of $[NH_4Cl]$, all the NH_4^+ ions provided by NH_4Cl as due to common ion effect, dissociation of NH_4OH will be suppressed.

$$-\log[OH^-] = -\log 1.8 \times 10^{-5} + \log\frac{[NH_4^+]}{[0.05]}$$

$$\therefore [NH_4^+] = 0.067\,M \text{ or } [NH_4Cl] = 0.067\,M$$

7.(B)



SECTION - 3

8.(B)

$$NaNO_2 + FeSO_4 + 3H_2SO_4 \rightarrow Fe_2(SO_4)_3 + NaHSO_4 + H_2O + NO$$

$$HNO_3 + P_4O_{10} \rightarrow HPO_3 + N_2O_5$$

$$Cu + HNO_3(\text{Conc.}) \rightarrow Cu(NO_3)_2 + NO_2 + H_2O$$

$$NH_4NO_3 \xrightarrow{\Delta} N_2O + 2H_2O$$

9.(C)

(P) $[Mn(NH_3)_6]^{2+}$ $NH_3 \rightarrow WFL$

Electronic configuration $\Rightarrow t_{2g}^3\, e_g^2$

$$\mu = \sqrt{5(5+2)} = \sqrt{35}\text{ B.M.}$$

(Q) $[Co(ox)_3]^{3-}$ $\rightarrow SFL$

Electronic configuration $\Rightarrow t_{2g}^6\, e_g^0$

$$\mu = 0$$

(R) $[Fe(H_2O)_6]^{2+}$ $H_2O \rightarrow WFL$

Electronic configuration $\Rightarrow t_{2g}^4\, e_g^2$

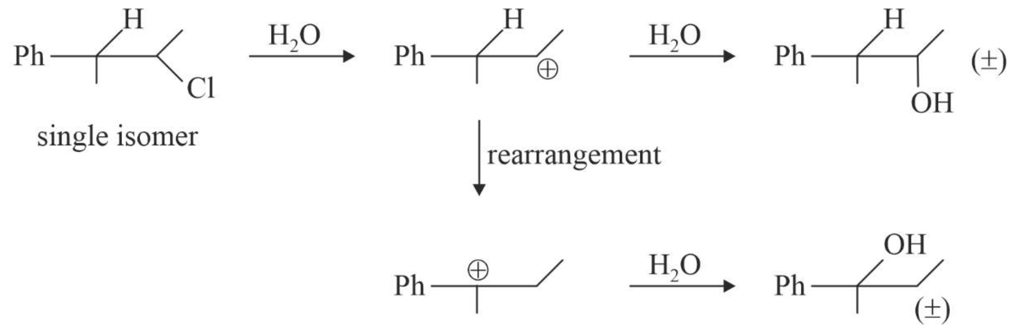
$$\mu = \sqrt{4(4+2)}\text{ B.M.} = \sqrt{24}\text{ B.M.}$$

(S) $[Co(H_2O)_6]^{2+}$ $H_2O \rightarrow WFL$

Electronic configuration $\Rightarrow t_{2g}^5\, e_g^2$

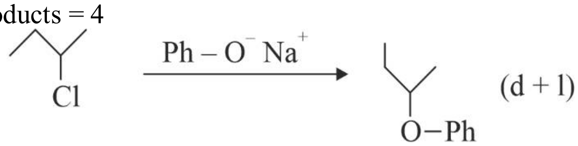
$$\mu = \sqrt{3(3+2)}\text{ B.M.} = \sqrt{15}\text{ B.M.}$$

10.(C) **(P)**



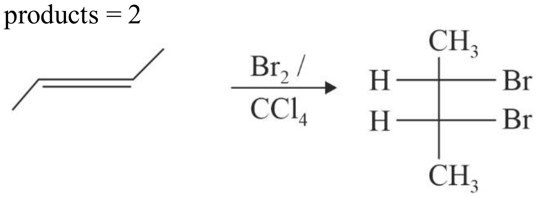
No. of products = 4

(Q)



No. of products = 2

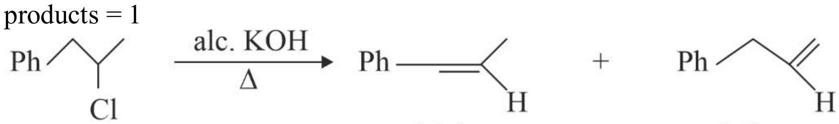
(R)



Meso

No. of products = 1

(S)

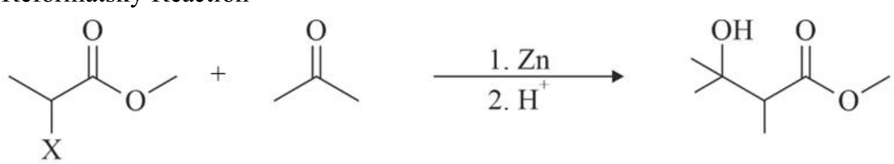


Major
(cis+trans)

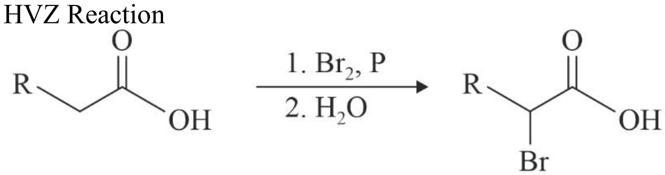
Minor

No. of products = 3

11.(C) Reformatsky Reaction

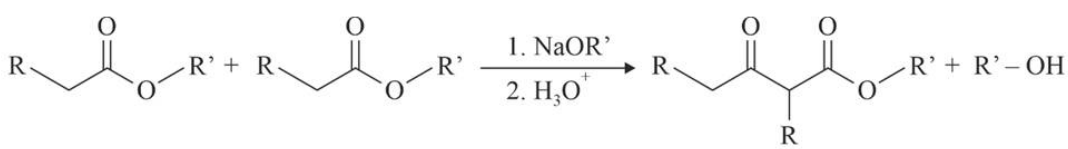


HVZ Reaction



Carboxylic acid α-Bromo carboxylic acid

Claisen condensation reaction



Ester

Beta-keto ester

Alcohol

Gabriel-phthalimide synthesis-preparation of primary amine

SECTION - 4

1.(434) X is trimethyl silyl chloride $(CH_3)_3SiCl$

Molar mass of X = 108.5

Mass of X required with 50% yield $= 2\times2\times108.5\,\text{gm}$

2.(1) $Z = \frac{(V_m)_{real}}{(V_m)_{ideal}} = 0.8$

$Z = 0.8$

$PV_m = ZRT$

$30\times V_m = 0.8\times0.082\times456$

$V_m = 1\,L$

3.(133) $\ln\frac{k_2}{k_1} = \frac{E_a}{R}\left[\frac{T_2 - T_1}{T_2T_1}\right]$

$$\ln\frac{2\times10^{-3}}{5\times10^{-5}} = \frac{E_a}{R}\left[\frac{500-450}{500\times450}\right]$$

$$\ln 40 = \frac{E_a}{8}\times\frac{50}{500\times450}$$

$$\frac{500\times450\times8\times3.7}{50} = E_a$$

$$133200 = E_a$$

$$133.2\text{kJ} = E_a$$

4.(432) $P_A = \frac{1\times R\times300}{V_0}$

$$P_B = \frac{1\times R\times300}{2V_0} = \frac{150R}{V_0}$$

$$W_{AB} = -nRT\ln\frac{V_2}{V_1} = -1\times R\times300\ln\frac{2V_0}{V_0} = -300R\ln2 = -210R$$

For process B to C

$$\left(\frac{150R}{V_0}\right)^{1-1.5} T_B^{\frac{3}{2}} = \left(\frac{150R}{4V_0}\right)^{1-1.5} T_C^{\frac{3}{2}}$$

$$300^{\frac{3}{2}} = \left(\frac{1}{4}\right)^{\frac{-1}{2}} T_C^{\frac{3}{2}}$$

$$300 = \left(\frac{1}{4}\right)^{\frac{-1}{3}} T_C = T_C$$

$$\frac{300}{(4)^{\frac{1}{3}}} = T_C$$

$$189K = T_C$$

$$W_{BC} = \frac{-nR(T_2 - T_1)}{\gamma - 1} = \frac{-1 \times R[189 - 300]}{\frac{3}{2} - 1} = -222R$$

$$\text{Total } W_{AC} = -210R - 222R = -432R$$

5.(12)

$$\Delta G°_r = \Delta H°_r - T\Delta S°_r$$

$$\Delta G°_r = (80 - 120) \times 10^3 - 300 \times (170 - 210)$$

$$\Delta G°_r = -40 \times 10^3 + 12 \times 10^3$$

$$\Delta G°_r = -28 \times 10^3 \, J$$

$$\Delta G°_r = -RT \ln k$$

$$-28 \times 10^3 = -RT \ln k$$

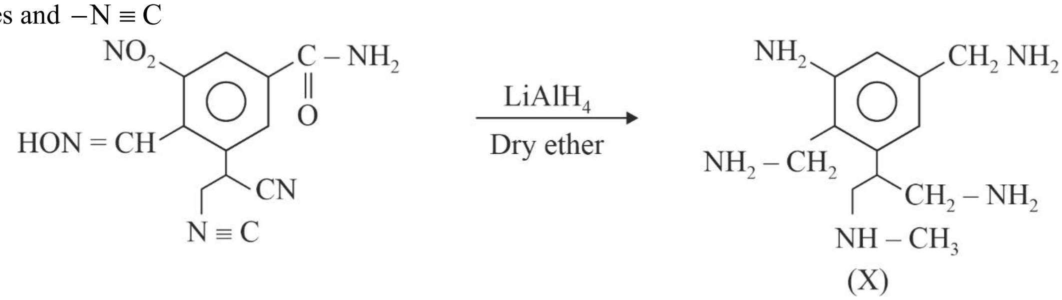
$$28 \times 10^3 = 8 \times 300 \ln k$$

$$11.666 = \ln k$$

6.(6)

$LiAlH_4$ can reduce $-NO_2, -\underset{\underset{O}{\|}}{C}-NH_2, -C \equiv N$

Oximes and $-N \equiv C$



MATHEMATICS

SECTION - 1

1.(ABC) **(A)** $^7C_3 \cdot 1 + {}^7C_2 \cdot 2 + {}^7C_1 \cdot 1 = 84$

(B) $^7C_3 \cdot \underline{|3} = 210$

(C) $^7C_3 = 35$

2.(ABD)

If $\Delta \neq 0$

$h^2 = ab \Rightarrow$ Parabola

$h^2 < ab \Rightarrow$ Ellipse

$h^2 > ab \Rightarrow$ Hyperbola

If a = 2, b = 1, c = 0

$\sqrt{2x} + \sqrt{y} = 1 \quad \Rightarrow 4x^2 + y^2 - 4xy - 4x - 2y + 1 = 0$ (Parabola)

If $a = 1, b = 2, c = 2$

$\sqrt{x} + \sqrt{2y} = \sqrt{2x+1} \quad \Rightarrow x^2 + 4y^2 - 12xy + 2x - 4y + 1 = 0$ (Hyperbola)

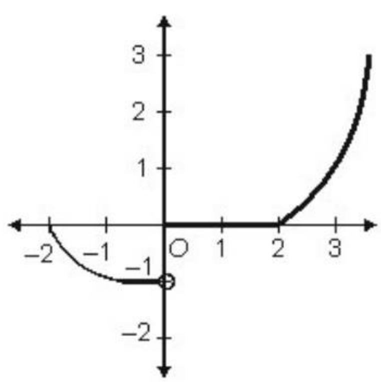
If $a = 0, b = 1, c = 1$

$\sqrt{y} = \sqrt{x+1}$ (line)

3.(ABC)

Here $f(x) = \begin{cases} x^2 + 2x, & x < 0 \\ x^2 - 2x, & x \geq 0 \end{cases}$

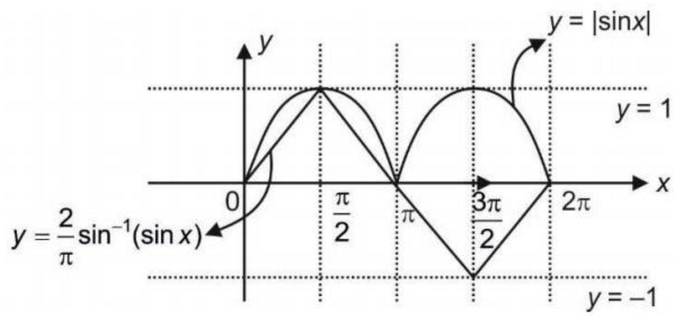
The graph of $g(x)$ is given as



$$\therefore g(x) = \begin{cases} x^2 + 2x, & -2 \leq x < -1 \\ -1, & -1 \leq x < 0 \\ 0, & 0 \leq x \leq 2 \\ x^2 - 2x, & 2 < x \leq 3 \end{cases}$$

SECTION - 2

4.(A)



$$\Rightarrow A_1 = \left(2-\frac{\pi}{2}\right)+2+\frac{\pi}{2}=4$$

$$A = 8A_1 = 8\times 4 = 32$$

$$\Rightarrow p = 32, q = 0 \Rightarrow p+q = 32$$

5.(C) Tetrahedron has one vertex, say O as (0, 0, 0)

$OA: y+z=0$ and $x+z=0$

$$\Rightarrow \frac{x-0}{1}=\frac{y-0}{1}=\frac{z-0}{-1} \qquad \ldots(1)$$

$BC: x+y=0$ and $x+y+z=\sqrt{6}$

$$\Rightarrow \frac{x-0}{1}=\frac{y-0}{-1}=\frac{z-\sqrt{6}}{0} \qquad \ldots(2)$$

Let PQ be the shortest distance between OA and BC

$$PQ = \frac{\left(\sqrt{6}\hat{k}\right)\cdot\left[(\hat{i}+\hat{j}-\hat{k})\times(\hat{i}-\hat{j})\right]}{\left|(\hat{i}+\hat{j}-\hat{k})\times(\hat{i}-\hat{j})\right|}=2$$

6.(A) X = bus A is late

Y = bus B is late

$$P(X)=\frac{1}{5}, P(Y/X)=\frac{9}{10}, P(Y)=\frac{7}{25}$$

$$\Rightarrow P(X\cap Y)=\frac{9}{10}\times P(X)=\frac{9}{10}\times\frac{1}{5}=\frac{9}{50}$$

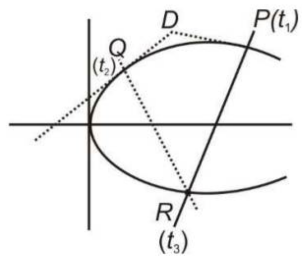
$$P(X\cup Y)=P(X)+P(Y)-P(X\cap Y)$$

$$=\frac{1}{5}+\frac{7}{25}-\frac{9}{50}=\frac{3}{10}$$

$$\therefore P(X'\cap Y')=1-\frac{3}{10}=\frac{7}{10}$$

$$P(X/Y)=\frac{P(X\cap Y)}{P(Y)}=\frac{9/50}{7/25}=\frac{9}{14}$$

7.(B)



$t_1t_2 = 2$ and $t_1 + t_2 + t_3 = 0,\ a = 2 \quad \Rightarrow \quad D = (a\,t_1t_2, a(t_1+t_2)) = (2a, -at_3)$

Let circumcenter of ΔDPQ is $O(h,k)$

$O(h,k)$ will be midpoint of RD

$\therefore h = \dfrac{at_1t_2 + at_3^2}{2} \Rightarrow h = 2 + t_3^2$

and $k = \dfrac{a(t_1+t_2)+2at_3}{2} \Rightarrow k = t_3$

$\therefore h = 2 + k^2$

locus is $y^2 = x - 2$

SECTION - 3

8.(A)

(P) $A = \begin{bmatrix} 2 & 5 & 10 \\ 5 & 8 & 13 \\ 10 & 13 & 18 \end{bmatrix} \Rightarrow$ Symmetric and $|A| = 0$

(Q) $A = \begin{bmatrix} 1 & 3^{-1} & 3^{-2} \\ 3^1 & 1 & 3^{-1} \\ 3^2 & 3^1 & 1 \end{bmatrix} \Rightarrow |A| = 0$

(R) $A = \begin{bmatrix} 0 & -3 & -8 \\ 3 & 0 & -5 \\ 8 & 5 & 0 \end{bmatrix} \Rightarrow$ Skew symmetric and $|A| = 0$

(S) $A^2 = A\,\&\, "|A| = 0"$

9.(C)

(P) $\displaystyle\sum_{i=1}^{n} \alpha_i^k = \begin{cases} 0 & \text{if } \dfrac{k}{n} \notin I \\ n & \text{if } \dfrac{k}{n} \in I \end{cases}$

So, $\displaystyle\sum_{i=1}^{7} \alpha_i^k \neq 0$ then $\dfrac{k}{7}$ is an integer

(Q) Number of common roots = HCF of 20 and 30 = 10

(R) Roots of $z^{19}=1$ are

$e^{\left(\frac{2k\pi i}{19}\right)}; k=0,1,...,18$

If $\frac{2k\pi}{19} \in \left(\pi, \frac{3\pi}{2}\right)$

$\Rightarrow k \in \left(\frac{19}{2}, \frac{57}{4}\right)$

$\Rightarrow k = 10, 11, 12, 13$ and 14 (Total 5 roots)

(S) Roots of $z^{14}=1$ are $e^{\frac{2k\pi i}{14}}$

$\therefore \text{Re}(z) > 0, \text{Im}(z) > 0$

$\Rightarrow 0 < \frac{2\pi k}{14} < \frac{\pi}{2} \Rightarrow k \in \left(0, \frac{7}{2}\right)$

$k = 1, 2, 3$

$\Rightarrow \lambda = \cos\frac{2\pi}{14} \cdot \cos\frac{4\pi}{14} \cdot \cos\frac{6\pi}{14} = \frac{1}{8}$

10.(C)

(P) $\left(x^3 - \frac{1}{x^2}\right)^n$

$T_{r+1} = {}^nC_r (x^3)^{n-r} \left(\frac{-1}{x^2}\right)^r$

$3n - 5r = 5 \Rightarrow r = \frac{3n-5}{5} = p$ (say)

$3n - 5r = 10$; $r = \frac{3n-10}{5} = p - 1 \quad \therefore {}^nC_p = {}^nC_{p-1}$

$\Rightarrow \quad n - p = p - 1$; $n + 1 = 2\left(\frac{3n}{5} - 1\right) \quad \Rightarrow \quad n = 15$

(Q) Number of non-negative integral solutions of

$\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 = 3$

${}^{3+3}C_3 = {}^6C_3 = 20$

(R) $k = \left[\frac{(n+1)|x|}{|x|+1}\right] = 6$ $\quad$ T_6 and T_7 both are greatest terms

(S) $\left(\frac{-1}{\sqrt{2}}\right)^n = \left(\frac{1}{3^{5/3}}\right)^{\log_3 8} = \frac{1}{(3^5)^{\log_3 2}} = 2^{-5}$

$\frac{n}{2} = 5$

$\Rightarrow n = 10$

11.(C) **(P)** $V=[\vec{a}\,\vec{b}\,\vec{c}]=2$

$V_1=[2(\vec{a}\times\vec{b})3(\vec{b}\times\vec{c})(\vec{c}\times\vec{a})]$

$=6[(\vec{a}\times\vec{b})(\vec{b}\times\vec{c})(\vec{c}\times\vec{a})]=6[\vec{a}\ \vec{b}\,\vec{c}]^2=6(2)^2=24$

(Q) $V=[\vec{a}\,\vec{b}\,\vec{c}]=5$

$V_1=[3(\vec{a}+\vec{b}),(\vec{b}+\vec{c}),2(\vec{c}+\vec{a})]$

$=6[(\vec{a}+\vec{b}),(\vec{b}+\vec{c}),(\vec{c}+\vec{a})]$

$=6\cdot 2[\vec{a}\ \vec{b}\,\vec{c}]=60$

(R) $A=\frac{1}{2}\left|\vec{a}\times\vec{b}\right|=20$

$A_1=\frac{1}{2}\left|(2\vec{a}+3\vec{b})\times(\vec{a}-\vec{b})\right|$

$=\frac{1}{2}\left|(3\vec{b}\times\vec{a}-2\vec{a}\times\vec{b})\right|$

$=\frac{1}{2}5\left|\vec{b}\times\vec{a}\right|=5(20)=100$

(S) $A=\left|\vec{a}\times\vec{b}\right|=30$

$A_1=\left|(\vec{a}+\vec{b})\times\vec{a}\right|=\left|\vec{b}\times\vec{a}\right|=30$

SECTION - 4

1.(14) $2y^2-2(\cos x+\sin x)y+2\sin x\cos x\le 0$

$\Rightarrow (y-\sin x)(y-\cos x)\le 0$

But $x\in\left(0,\frac{\pi}{4}\right)$ $\quad\therefore \cos x>\sin x$

$\Rightarrow \sin x\le y\le\cos x$

$\Rightarrow \sin x\le f(x)\le\cos x$

$\therefore k_1=\int_0^{\frac{\pi}{4}}\cos x\,dx=\frac{1}{\sqrt{2}}$,

$k_2=\int_0^{\frac{\pi}{4}}\sin x\,dx=\left(1-\frac{1}{\sqrt{2}}\right)$

$10k_1=5\sqrt{2}\Rightarrow[10k]=7$

$10k_2=10-5\sqrt{2}\Rightarrow[10k_2]=2$

$\therefore[10k_1]\cdot[10k_2]=14$

2.(2) $f(x)=\int_0^1 e^{x+t} f(t)dt+e^x$

$f(x)=e^x\left(1+\int_0^1 e^t f(t)dt\right)$. Let $\int_0^1 e^t f(t)dt=A$

$\Rightarrow f(x)=e^x(1+A)\Rightarrow f(t)=e^t(1+A)$

$\Rightarrow A=\int_0^1 e^t\cdot e^t(1+A)dt$

$\Rightarrow A=(1+A)\left(\dfrac{e^2-1}{2}\right)$

$\Rightarrow \dfrac{A}{1+A}=\dfrac{e^2-1}{2}$

$\Rightarrow \dfrac{1}{1+A}=\dfrac{e^2-3}{2}$

$\Rightarrow A+1=\dfrac{2}{3-e^2}\Rightarrow f(x)=\left(\dfrac{2}{3-e^2}\right)e^x\Rightarrow \left(3-e^2\right)f(0)=-2$

3.(1) $a_{n+1}-a_n=(a_n-1)^2>0 \qquad \forall\, n\in N$

$\Rightarrow \quad a_n$ is increasing sequence

$a_{n+1}-1=a_n^2-a_n$

$\dfrac{1}{a_n-1}-\dfrac{1}{a_n}=\dfrac{1}{a_{n+1}-1}$

$\sum_{n=1}^{\infty}\dfrac{1}{a_n}=\sum_{n=1}^{\infty}\left(\dfrac{1}{a_n-1}-\dfrac{1}{a_{n+1}-1}\right)$

$=\dfrac{1}{a_1-1}-\left(\lim_{n\to\infty}\dfrac{1}{a_{n+1}-1}\right)=1$

$\therefore \lim_{n\to\infty} a_{n+1}\to\infty \qquad$ (increasing sequence)

4.(4) According to question

$(z-\alpha_1)(z-\alpha_2)...(z-\alpha_6)$

$=1+2z+3z^2+...+7z^6$

Now take log on both side

$\log(z-\alpha_1)+\log(z-\alpha_2)+...\log(z-\alpha_6)$

$= \ log\left(1+2z+3z^2 \ldots+7z^6\right)$

Now diff. it and put $z = 1$

We get, $\sum_{i=1}^{6}\frac{1}{1-\alpha_i}=4$

5.(6) Here equation of line is

$\frac{x-2}{1}=\frac{y+3}{2}=\frac{z-4}{-3}=\lambda$ say ...(i)

Hence any point on the line (i) will be $p(\lambda+2,2\lambda-3,-3\lambda+4)$

Given line (i) intersect the given plane

$2x+3y-z=13$ at the point P

$\therefore 2(\lambda+2)+3(2\lambda-3)+3\lambda-4=13$

$\Rightarrow 11\lambda-9=13$

$\Rightarrow 11\lambda=22 \quad \Rightarrow \lambda=2$

$\therefore P=(4,1,-2)$

Also, equation (i) intersects the YZ - plane at Q

i.e., $x=0$

$\therefore \quad \lambda+2=0 \quad \Rightarrow \quad \lambda=-2$

$\therefore \quad Q\equiv(0,-7,10)$

$\therefore \quad PQ=\sqrt{(4-0)^2+(1+7)^2+(-2-10)^2}$

$=\sqrt{16+64+144}$

$=\sqrt{224}=\sqrt{16\times14}=4\sqrt{14}=a\sqrt{b}$

$\therefore \quad \frac{a+b}{3}=\frac{4+14}{3}=6$

6.(4) $N^r=\sum_{k=0}^{4}{}^4C_{4-k}\,.{}^nC_{r+k} \ = {}^{n+4}C_{r+4}$

$D^r=\sum_{k=0}^{3}{}^3C_{3-k}\,.{}^nC_{r+k} \ = {}^{n+3}C_{r+3}$